

Technical Comments

Comment on "Similarity Analyses via Group Theory"

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I WOULD like to thank Moran and Gaggioli for their comments¹ on my paper.² The intention in Ref. 2 was to reduce Eqs. (1–6) via group theory directly to a system of ordinary differential equations. Therefore, as suggested,¹ to obtain the results presented in Eqs. (19) and (20), a two-parameter group of transformations should be used; specifically, the first line of Eq. (18) should be changed to read

$$x^* = a^\alpha x - Cb \quad t^* = a^\alpha t + Bb \quad y^* = a^\beta y$$

The subsequent remarks in my original article for $\alpha = 0$ should be omitted. Equation (13) in Ref. 1 should include

$$\bar{\rho}_1 = a^{\alpha(1-2A)} \rho_1 \quad \bar{U}_1 = U_1 \quad \bar{V} = a^{\alpha(A-1)} V \\ \bar{T}_w = T_w \quad \bar{T}_1 = T_1$$

If desired, one-parameter transformation groups could be used to obtain a system of partial differential equations for systems with three or more independent variables. Fewer restrictions are generally imposed on the boundary data than would be imposed if two- (or more) parameter transformations were used.^{1,3–6} Successive use of one-parameter transformations would lead to a system of ordinary differential equations.

It should be noted that the transformations in Eq. (7), Ref. 2 are only a special case of the continuous two-parameter group of transformations

$$x_i^* = g_i(x_1, \dots, x_m; a, b) \quad y_r^* = h_r(y_1, \dots, y_n; a, b)$$

Therefore, because of the limited types of groups considered, the similarity solutions presented are not claimed to be the only ones obtainable. As recently shown by Moran and Gaggioli,⁷ it is not necessary to choose the group of transformations in advance. For one-parameter groups, Ref. 7 describes how the appropriate classes of groups are deduced by considering the system of partial differential equations and boundary conditions. The similarity variables are then systematically derived. Similar procedures for multiple-parameter groups of transformations are discussed in Ref. 8.

Useful results were contained in Ref. 2 by requiring the boundary conditions as well as the partial differential equations to be constant conformally invariant under the continuous groups of transformations. However, theoretical proof of this approach has not yet been established.⁷

References

- 1 Moran, M. J. and Gaggioli, R. A., "Similarity Analyses via Group Theory," *AIAA Journal*, Vol. 6, No. 10, Oct. 1968, pp. 2014–2016.
- 2 Gabbert, C. H., "Similarity for Unsteady Compressible Boundary Layers," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1198–1200.
- 3 Morgan, A. J. A., "Discussion of Possible Similarity Solu-

tions of the Laminar, Incompressible, Boundary-Layer Equations," *Transactions of the ASME*, Vol. 80, 1958, pp. 1559–1562.

⁴ Hansen, A. G., *Similarity Analyses of Boundary Value Problems in Engineering*, Prentice-Hall, Englewood Cliffs, N. J., 1964, Chap. 4.

⁵ Manohar, R., "Some Similarity Solutions of Partial Differential Equations of Boundary Layer," Tech. Summary Rept. 375, Jan. 1963, Mathematics Research Center, U.S. Army, University of Wisconsin, Madison.

⁶ Ames, W. F., *Nonlinear Partial Differential Equations in Engineering*, Academic Press, New York, 1965, pp. 133–144.

⁷ Moran, M. J. and Gaggioli, R. A., "Reduction of the Number of Variables in Systems of Partial Differential Equations, with Auxiliary Conditions," *SIAM Journal of Applied Mathematics*, Vol. 16, No. 1, Jan. 1968, pp. 202–215.

⁸ Moran, M. J., "A Unification of Dimensional and Similarity Analysis via Group Theory," Ph.D. thesis, University of Wisconsin, Madison, Aug. 1967, Chaps. 4 and 7.

Erratum: "Boundary-Layer Transition on Ablating Cones at Speeds up to 7 km/sec"

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SUBSEQUENT to the publication of the above-titled article, an error in the computation of the tip ablation was discovered. The constant k in Eqs. (8) and (9) should be equal to 1.25×10^{-3} ; it was incorrectly given and used as 1.86×10^{-3} .

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Erratum: "Symptomatic Behavior of an Electric Arc with a Superimposed Flow"

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THE editors regret that, in the above article, Fig. 2 on p. 1475 (the five sequential photographs from a high speed movie of argon arc in restrike mode of operation) was printed upside down.

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